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IMPROVED STRESS INTENSITY SOLUTIONS DEVELOPED FOR THE MULTIPLE SITE DAMAGE SCENARIO

Two Unequal Through Cracks on Either Side of an Open Hole, Multiple Through Cracks, and Through Cracks Approaching an Open Hole

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Aerospace Vehicles Division

OCTOBER 2014
Interim Report

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14. ABSTRACT
This report documents work performed to develop new, improved stress intensity factor (K) solutions for geometries that may be useful as components of a more complex solution for the multi-site damage geometry. These solutions include:

- Two, unequal through cracks on either side of an open hole in an infinite plate
- Multiple through cracks along a plane in an infinite plate
- Through crack approaching an open hole in an infinite plate.

The solutions were developed using existing handbook solutions (where possible) and were modified as needed based on K-solutions obtained from finite element (FE) validation models. A wide range of geometric parameters, compatible with typical hole spacing values used in actual aircraft structures, were used for the FE models.

The improved solutions for all three cases are shown to be within 3 percent of the FE model results obtained as a part of this effort. The majority of the cases are within 1 percent of these results.

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1 Introduction

Stress intensity factor (K) corrections for most common geometries are available in several published handbooks [1-3]. In most cases, the solutions are considered to be exact, but in some cases, the accuracy is limited, and can often be improved by the use of current finite element modeling (FEM) tools such as StressCheck[4]. In this report, the normalize stress intensity parameter, $\boldsymbol{\beta}$, will be used as shown below.

$$\boldsymbol{K = \sigma_{ref} \sqrt{\pi x}\, \beta_{(x)}} \tag{1}$$

Where:

$\boldsymbol{\sigma_{ref}}$: reference stress used to determine K

$\mathbf{x}$: crack length of interest

As indicated in Equation 1, the beta value is a function of the crack length, but it also accounts for geometry effects on the local stress field near the crack tip. Existing solutions for common crack geometries may be useful as components of a compounded solution for the more complex solution, such as the multi-site damage geometry, shown in Figure 1.

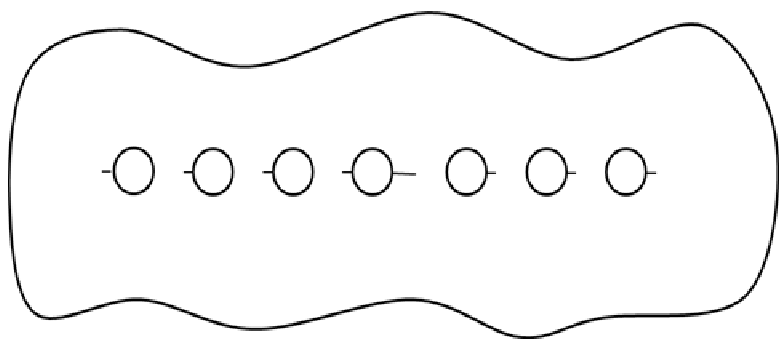

Figure 1: Typical Multiple Crack Assumption for the MSD Scenario

This MSD problem assumes the existence of a primary crack on one side of a fastener hole with a shorter, secondary crack on the opposite side of the hole. Additional secondary cracks are assumed to exist on the opposite side of every other hole in a row for an infinite plate.

The following sections summarize work performed to develop new K-solutions that may be useful in the development of a 2-D compounded solution for the MSD crack geometry.

1.1 Two Un-Equal Through Cracks on Either Side of an Open Hole in an Infinite Plate

To predict the growth of these cracks (shown in Figure 2), it is necessary to account for crack interaction. Each crack has an effect on the local stress field as it grows, so the stress intensity factor at each crack tip is a function of the length of both cracks. The handbook solution by Murakami [3], was reported to be accurate within 10 percent for all crack lengths, but assumed to be within 5 percent for most crack length combinations. The handbook solution is given as:

$$\boldsymbol{\beta} = \boldsymbol{F}_{(\lambda)} \sqrt{\frac{\boldsymbol{R+0.5(C_1+C_2)}}{\boldsymbol{R+C_1}}} \tag{2}$$

Where:
C_1: crack length of interest
C_2: crack length on the opposite side of the hole
$F_{(\lambda)}$: beta solution for 2, symmetric cracks of length C_1 on each side of the hole[1]

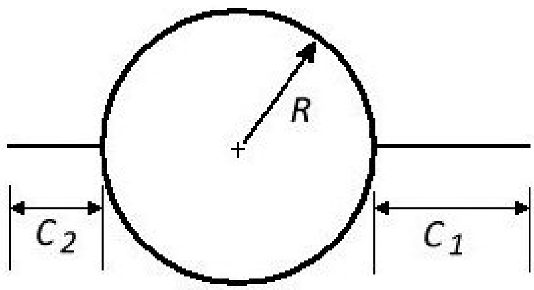

Figure 2: Un-Equal Cracks on Either Side of a Hole

The range of applicability for the handbook solution is given as: $0 \leq C_i/R \leq 1$. In this case, the subscript (i) refers to C_1 or C_2, and there is no restriction on which crack is longer. It is interesting to note that the form of the correction used to convert the double, symmetric through cracked hole solution is very similar to the Shah correction [5], that has been commonly used to convert double, symmetric, quarter elliptical cracked hole solutions to the corresponding single cracked case, as shown in Figure 3.

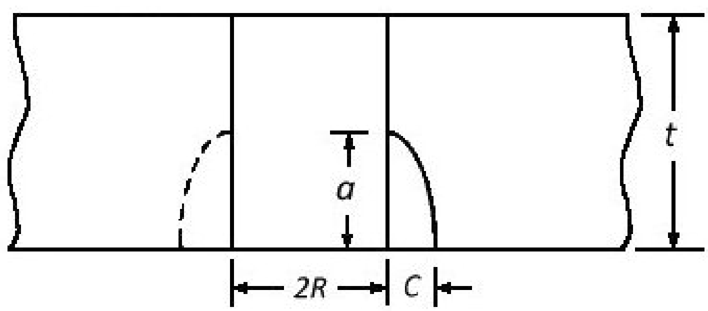

Figure 3: Symmetric Double/Single Corner Crack at a Hole in an Infinite Plate

The Shah correction for the single/double corner cracked case is given in Equation 3.

$$\boldsymbol{\beta_{Single\ Crack}} = \boldsymbol{\beta_{Double\ Crack}} \sqrt{\frac{\frac{4}{\pi} + \frac{a\,c}{2tR}}{\frac{4}{\pi} + \frac{a\,c}{tR}}} \tag{3}$$

[1] The handbook references a closed-form solution for the double, symmetric crack geometry as a function of $\lambda = C_1/R$.

After setting a/t = 1 (for a through crack), and applying some algebra, the Shah correction for a through crack is given below in Equation 4.

$$\beta_{Single\ Crack} = \beta_{Double\ Crack}\sqrt{\frac{8R+\pi C}{8R+2\pi C}} \tag{4}$$

An issue of some concern is the fact that Equations 2 and 4 are not the same as they should be when $C_2 = 0$ (single crack case) in Equation 2. To resolve this, FE analyses for a number of crack lengths were performed using StressCheck for single and double symmetric through cracks at an open hole in a very wide plate (W/D = 100). The resulting beta values for a gross remote stress reference are shown in Table 1.

Table 1: Beta Values for Single and Double Symmetric Through Cracks at Holes

C/R	Single Crack	Double Crack
0[3]	3.365	3.365
0.008	3.310	3.316
0.02	3.230	3.233
0.04	3.105	3.111
0.08	2.884	2.908
0.12	2.695	2.724
0.2	2.388	2.425
0.24	2.264	2.307
0.3	2.104	2.158
0.4	1.894	1.967
0.5	1.734	1.824
0.6	1.609	1.709
0.8	1.430	1.557
1	1.308	1.450
1.2	1.220	1.373
1.6	1.105	1.267
2	1.032	1.225
2.4	0.982	1.188
4	0.878	1.115
6	0.824	1.080
8	0.796	1.060
16	0.753	1.006

The results for the single crack were divided by the corresponding double crack beta values and compared to the handbook solution (with $C_2 = 0$) and the through crack Shah correction. The purpose of this comparison was to evaluate the accuracy of these closed-form double to single crack beta corrections. Both solutions were generally below the FEM results up to C/R = 4. The

[3] The zero crack length values are based on the reference stress concentration and the finite edge correction.

handbook solution was as much as 4 percent low and the Shah correction was better at 2.7 percent low.

An alternative solution for unequal through cracks at open holes is proposed that fits the FE results better for C2 = 0, C1/R < 2 and is less than 2% high (conservative) for all C1/R > 2.

$$\boldsymbol{\beta = F_\lambda \sqrt{\frac{1.645\,R+0.5(C_1+C_2)}{1.645\,R+C_1}}} \tag{5}$$

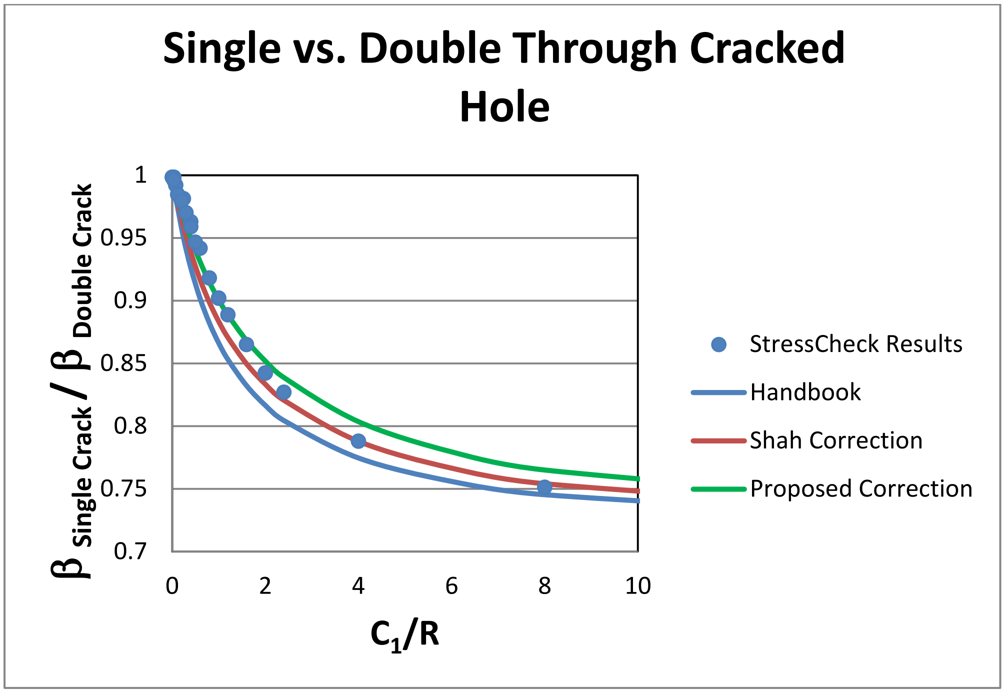

Figure 4: Correction for Single vs. Double Cracks at an Open Hole

The proposed solution was developed as a curve fit to the FE based data, and was biased to provide the best fit for the shorter crack lengths. Although it would have been possible to develop a more accurate fit to all of the data shown in Figure 4, the proposed solution also had to meet the following criteria:

The solution must converge to 1.0 as the crack lengths go to zero, and

The solution must converge to $\sqrt{0.5}$ as the crack lengths go to infinity.

These limits reflect the exact solutions for single and double symmetric through cracks at an open hole in an infinite plate. It is clear from examination that equations 2,4, and 5 meet the above requirements. In addition, it was desirable to use the same equation format as the handbook solution for the more general case of two, non-symmetric cracks at an open hole. The handbook solution is reported to be accurate within 10 percent for all crack lengths and 5 percent for most crack length combinations, where $C_i/R \leq 1$.

The new proposed solution (Equation 5) was then applied to the case of two, nonsymmetric through cracks at an open hole in an infinite plate. Comparisons with StressCheck FEM results

for several combinations of crack lengths indicated differences as high as 8 percent (for C_1/R = 6). When considering the use of this correction as part of a larger compounded solution, additional improvements are needed. For example, a 2 percent difference compounded over 3 solutions results in an overall difference of approximately 6 percent. Historically, differences as high as 5 percent have been acceptable for life prediction purposes. Therefore, it is important to make improvements to the solution to minimize these differences.

A set of curve fit solutions were developed as closed-form corrections required to resolve the differences between the proposed solution and FE results. The corrections were simplified by defining a primary (C_p) and secondary crack (C_s) on either side of the hole, where $C_s \leq C_p$. This was done so that a closed-form correction could be developed for each crack. Several crack length combinations were used for $0.4 \leq C_p/R \leq 6$ to determine the required corrections. The closed-form corrections are shown as lines overlaid on FE data points for each crack tip in Figure 5 and Figure 6. Corrections below $C_p/R = 0.3$ were linearly interpolated[4] to 1.0 as C_p/R approaches zero to allow the solutions to converge to the known solution. The closed-form, curve fit corrections for each crack tip are provided in equation form in Appendix A.

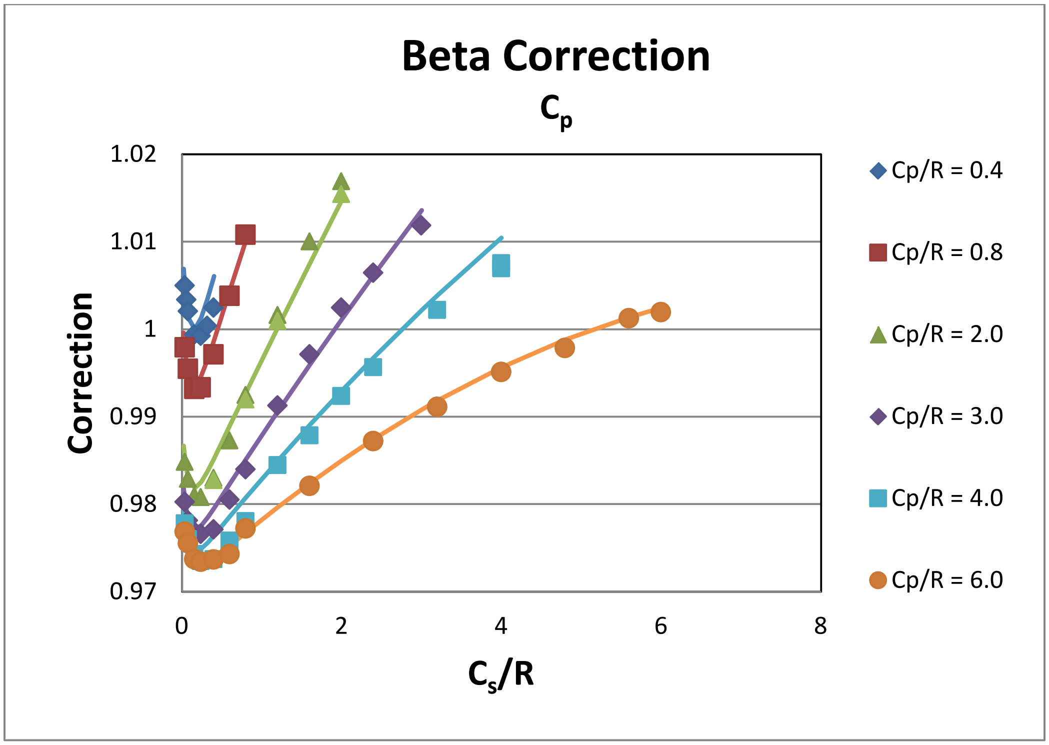

Figure 5: Beta Correction for C_p

[4] Test FE cases for $C_p/R = 0.2$ indicated good agreement with the data for $C_p/R = 0.4$, so the closed-form equation was used to $C_p/R = 0.3$, then linearly interpolated back to $C_p/R = 0$.

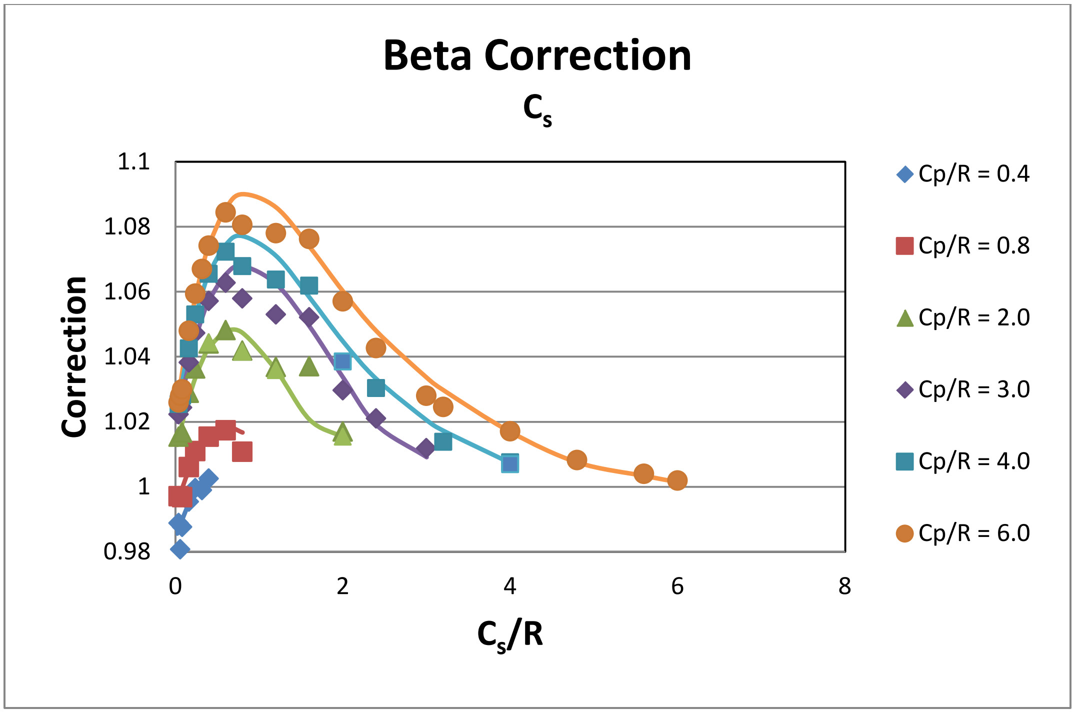

Figure 6: Beta Correction for C_s

These closed-form beta corrections have improved the accuracy of the proposed solution to within approximately 1 percent of the FEM results for $0.4 \leq C_p/R \leq 6$. Although not fully tested, all indications are that the linear interpolated solutions from $C_p/R = 0.3$ to 0.0 are well within 2 percent (probably closer to 1 percent).

In summary, the complete beta solution for this geometry requires the use of the proposed solution shown in Equation 5, and the additional application of the appropriate beta correction factor. When using the proposed solution, C_1 is always defined as the crack tip of interest, and C_2 is the crack length on the opposite side of the hole. The additional beta correction factors for the primary (C_p) and secondary (C_s) crack tip are determined using the closed form solutions provided in Appendix A for each crack tip, where $C_s \leq C_p$.

$$\boldsymbol{\beta_i = F_\lambda \sqrt{\frac{1.645\,R+0.5(C_1+C_2)}{1.645\,R+C_1}}\;(Beta\ Correction_i)} \quad (6)$$

1.2 Multiple Through Cracks Along a Plane in an Infinite Plate

Handbook solutions for multiple through cracks in an infinite plate (see Figure 7) are generally limited to cracks of equal length. A more general solution[5] is needed for the development of a compounded solution for the MSD geometry.

[5] Where, i > 2 in Figure 7.

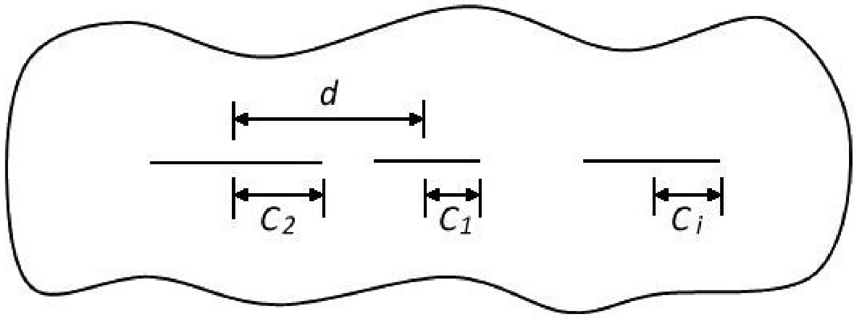

Figure 7: Multiple Through Cracks in an Infinite Plate

The first step in this case was to develop a solution for two cracks in an infinite plate. Two parameters were used to describe this geometry:

> $\mathbf{C_1/C_2}$, and
> $\mathbf{(C_1 + C_2)/d}$.

There are four crack tips to consider in this case. Two crack tips growing toward each other (inside tips), and two tips growing apart (outside tips). To simplify the solution, the crack tip of interest is defined as C_1, and the adjacent crack is defined as C_2.

Several combinations of crack lengths and spacing were modeled to obtain sufficient data for development of a closed-form solution. It is interesting to note that for any crack spacing (d), the beta solution for any crack tip was completely defined by the two parameters listed above.

Closed-form beta solutions for a remote gross tension reference stress were developed by curve-fitting the results for the FEMs for very large plates. The resulting equations are provided in Appendix B.

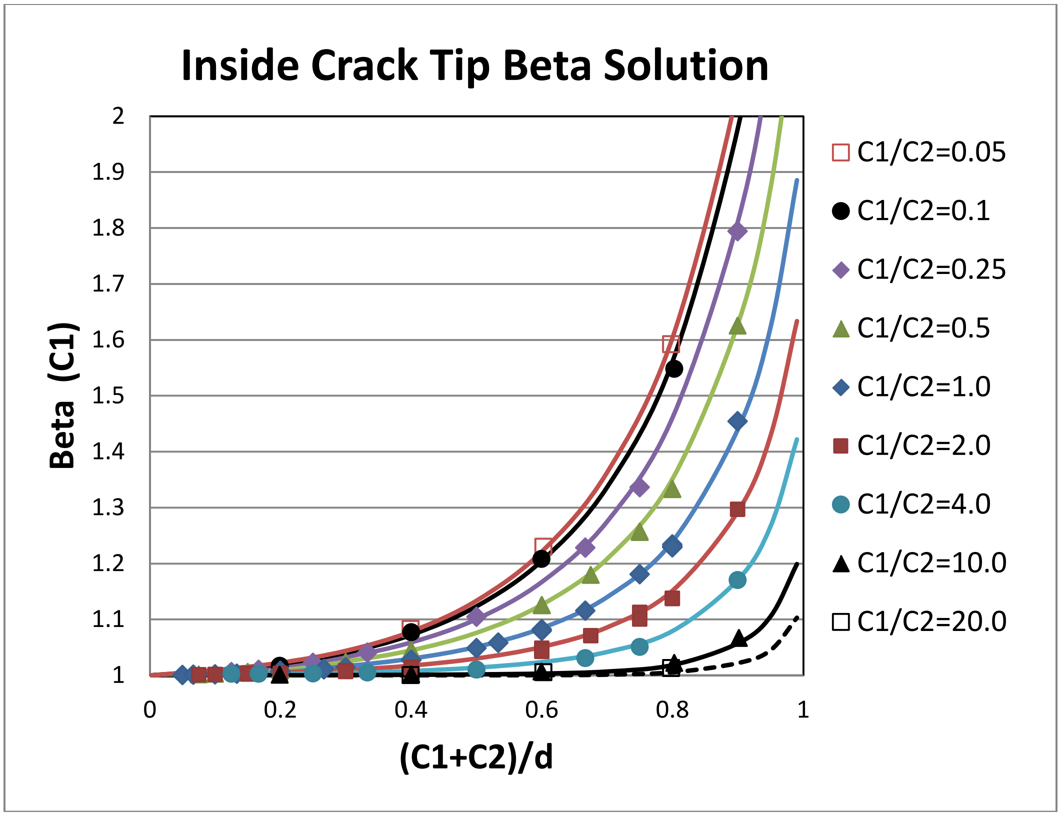

Figure 8: Beta Solution for the Inside Crack Tips

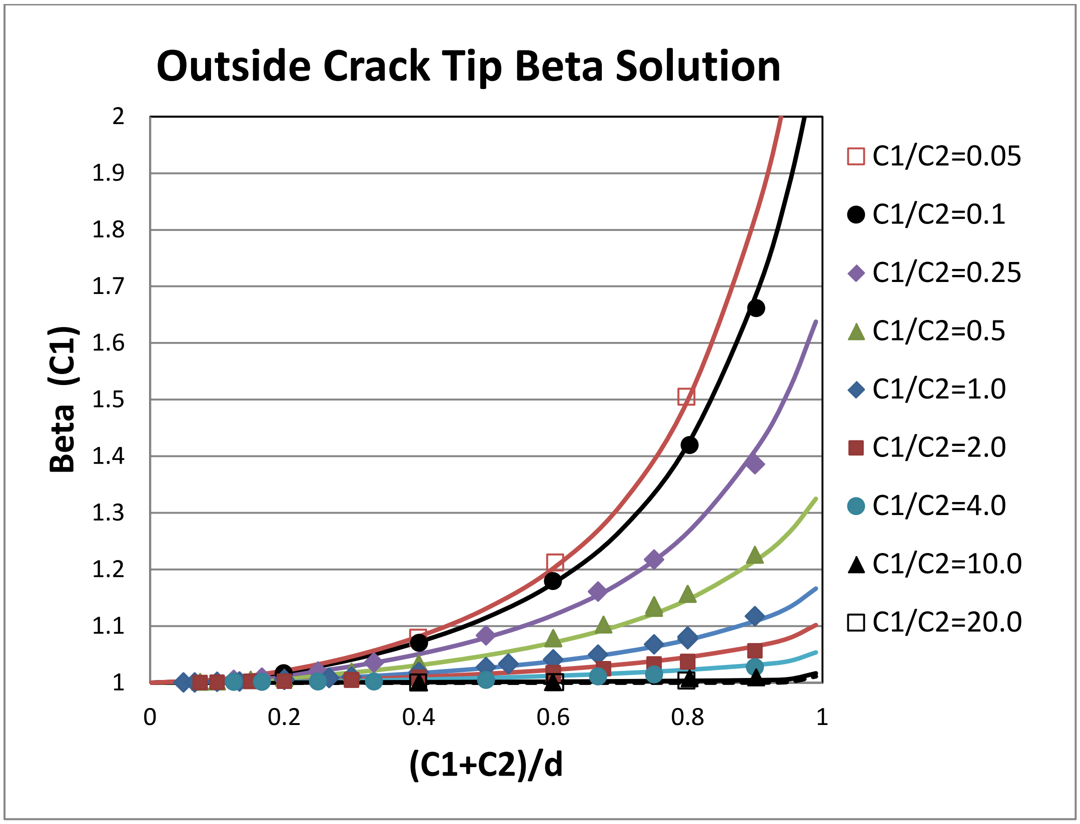

Figure 9: Beta Solution for the Outside Crack Tips

The two-crack solution was used to determine the solution for three-cracks (see Figure 6) using the method of compounding. The center crack was defined as C1 with cracks of equal length, C2, were placed on each side with crack spacing defined as d1 (left side) and d2 (right side). The solution for the center crack was calculated by applying the two-crack solution at each crack tip to account for the effect of the crack on either side. The product of the resulting solutions for each crack tip was compared to the FE solution for a number of crack lengths and spacings. A typical StressCheck p-version FEM is shown in Figure 9. The total plate width for these models was fifty times the distance between the outside crack tips of the right and left cracks, and the plate height was five times the width to approximate an infinite plate.

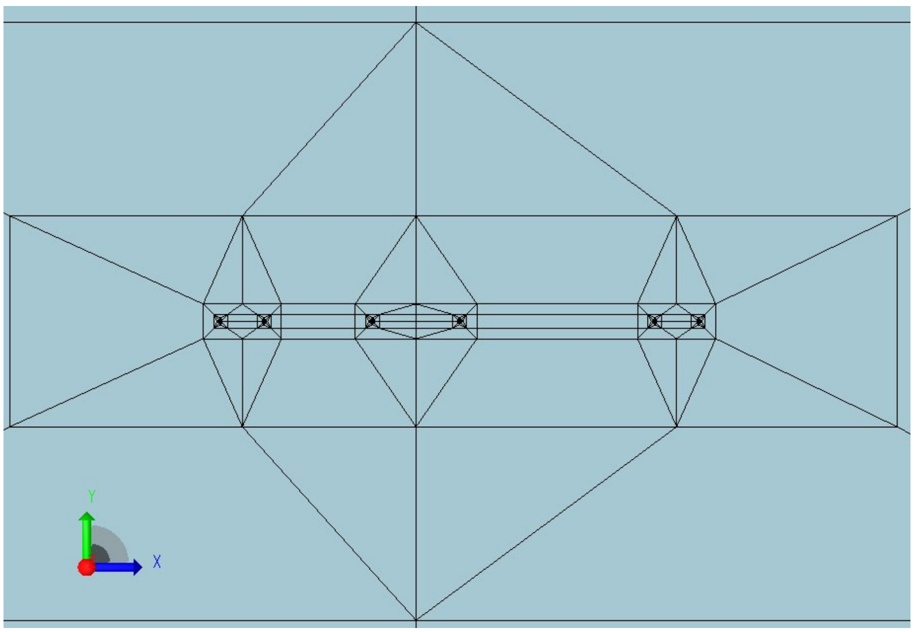

Figure 10: Typical StressCheck Model for Three Cracks

The FEM solutions were compared to the compounded, two crack beta solutions for the left and right crack tips of the center crack. The result is shown in Table 2, and Table 3 for the left (C_{11}) and right (C_{12}) crack tips, respectively.

Table 2: Beta Values for the Left Crack Tip

C1	C2	d1	d2	Beta 11	Bc11(inside)	Bc11(outside)	Bc11	%Diff
0.200	0.200	1.0	1.0	1.045	1.030	1.017	1.047	0.244
0.250	0.250	1.0	1.0	1.077	1.051	1.026	1.078	0.156
0.350	0.350	1.0	1.0	1.199	1.140	1.054	1.202	0.242
0.200	0.200	1.0	1.5	1.036	1.030	1.008	1.038	0.211
0.250	0.250	1.0	1.5	1.061	1.051	1.012	1.064	0.290
0.350	0.350	1.0	1.5	1.163	1.140	1.023	1.166	0.334
0.200	0.100	1.0	1.0	1.011	1.010	1.006	1.016	0.447
0.250	0.125	1.0	1.0	1.018	1.015	1.009	1.024	0.668
0.350	0.175	1.0	1.0	1.044	1.035	1.017	1.053	0.799
0.500	0.250	1.0	1.0	1.135	1.113	1.038	1.156	1.884
0.200	0.100	1.0	1.5	1.009	1.010	1.003	1.013	0.348
0.250	0.125	1.0	1.5	1.014	1.015	1.004	1.020	0.525
0.350	0.175	1.0	1.5	1.037	1.035	1.008	1.043	0.605
0.500	0.250	1.0	1.5	1.115	1.113	1.016	1.131	1.474
0.100	0.200	1.0	1.0	1.042	1.025	1.018	1.044	0.193
0.125	0.250	1.0	1.0	1.068	1.039	1.028	1.068	-0.047
0.175	0.350	1.0	1.0	1.149	1.087	1.053	1.145	-0.329
0.250	0.500	1.0	1.0	1.408	1.270	1.122	1.425	1.218
0.100	0.200	1.0	1.5	1.032	1.025	1.009	1.034	0.203
0.125	0.250	1.0	1.5	1.052	1.039	1.013	1.052	0.037
0.175	0.350	1.0	1.5	1.114	1.087	1.024	1.113	-0.092
0.250	0.500	1.0	1.5	1.318	1.270	1.048	1.331	1.025

Table 3: Beta-Values for the Right Crack Tip

C1	C2	d1	d2	Beta 12	Bc12(outside)	Bc12(inside)	Bc12	%Diff
0.200	0.200	1.0	1.0	1.045	1.017	1.030	1.047	0.244
0.250	0.250	1.0	1.0	1.077	1.026	1.051	1.078	0.156
0.350	0.350	1.0	1.0	1.199	1.054	1.140	1.202	0.242
0.200	0.200	1.0	1.5	1.026	1.017	1.014	1.031	0.501
0.250	0.250	1.0	1.5	1.044	1.026	1.020	1.047	0.320
0.350	0.350	1.0	1.5	1.101	1.054	1.043	1.099	-0.153
0.200	0.100	1.0	1.0	1.011	1.006	1.010	1.016	0.447
0.250	0.125	1.0	1.0	1.018	1.009	1.015	1.024	0.668
0.350	0.175	1.0	1.0	1.044	1.017	1.035	1.053	0.799
0.500	0.250	1.0	1.0	1.135	1.038	1.113	1.156	1.884
0.200	0.100	1.0	1.5	1.008	1.006	1.005	1.011	0.286
0.250	0.125	1.0	1.5	1.011	1.009	1.007	1.016	0.466
0.350	0.175	1.0	1.5	1.022	1.017	1.013	1.031	0.828
0.500	0.250	1.0	1.5	1.060	1.038	1.030	1.070	0.948
0.100	0.200	1.0	1.0	1.042	1.018	1.025	1.044	0.193
0.125	0.250	1.0	1.0	1.068	1.028	1.039	1.068	-0.047
0.175	0.350	1.0	1.0	1.149	1.053	1.087	1.145	-0.329
0.250	0.500	1.0	1.0	1.408	1.122	1.270	1.425	1.218
0.100	0.200	1.0	1.5	1.026	1.018	1.013	1.031	0.484
0.125	0.250	1.0	1.5	1.043	1.028	1.018	1.046	0.316
0.175	0.350	1.0	1.5	1.091	1.053	1.034	1.089	-0.147
0.250	0.500	1.0	1.5	1.214	1.122	1.076	1.208	-0.505

1.3 Through Cracks Approaching an Open Hole in an Infinite Plate

Handbook stress intensity solutions are available for this geometry, but are limited to a graphical format. A closed-form solution is far more desirable for use in a life prediction program.

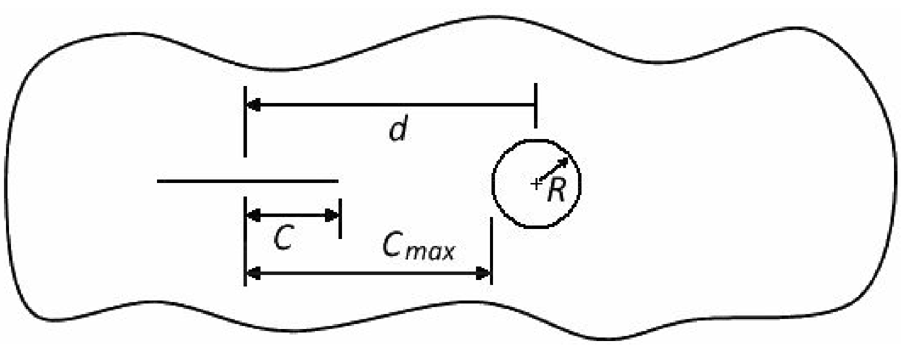

Figure 11: Through Crack Approaching a Hole

The parameters used for this solution are the same as may be found in the Tada handbook [2].

> **R/d,** and
> **C/C_{max}**.

The data used to develop closed-form solutions for each crack tip were obtained from a previous (2004) AFRL in-house effort [6] to characterize the effect of an open hole on an adjacent through crack. The finite element data[7] were spline fit to develop a table look-up solution for each crack tip. The tabular solutions are based on the remote gross reference stress for the infinite plate geometry, and are provided for the following parameters and associated limits:

> $\mathbf{0.0625 \le \boldsymbol{R/d} \le 0.97}$, and
> $\mathbf{0.0 \le C/C_{max} \le 1.0}$.

The tabular data and closed-form solutions are shown for the inside and outside crack tips in Figure 12 and Figure 13, respectively. The inside crack tip is defined as the crack tip growing toward the hole, and the closed-form beta solutions are provided in Appendix C.

[7] The FEMs in the previous AFRL effort used h-version elements.

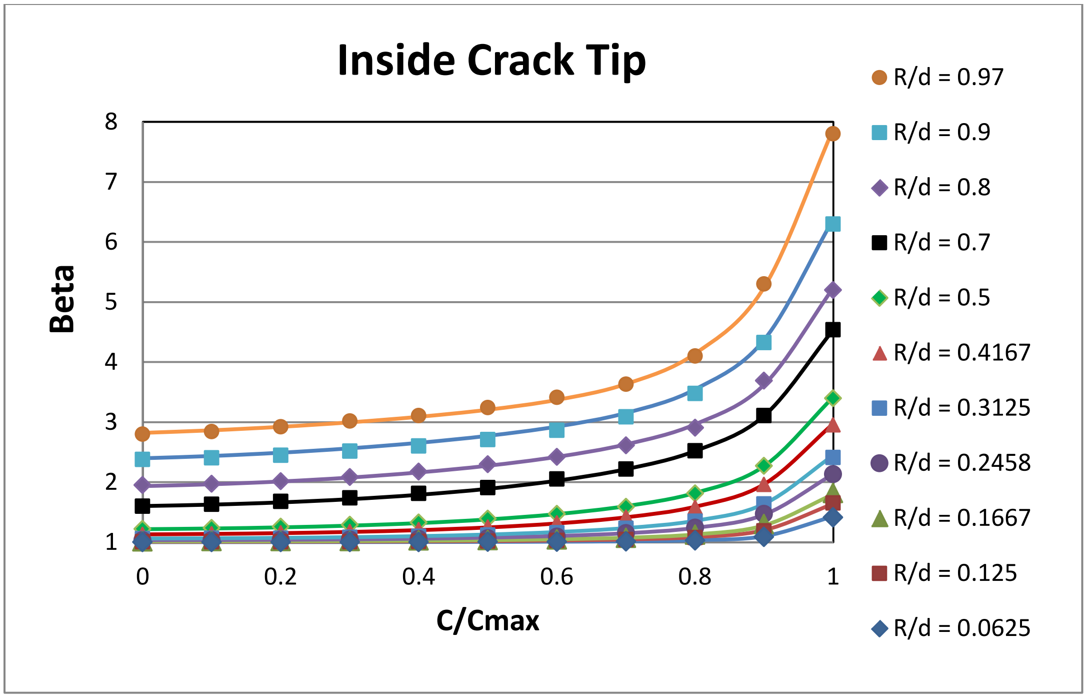

Figure 12: Beta Correction for the Inside Crack Tip

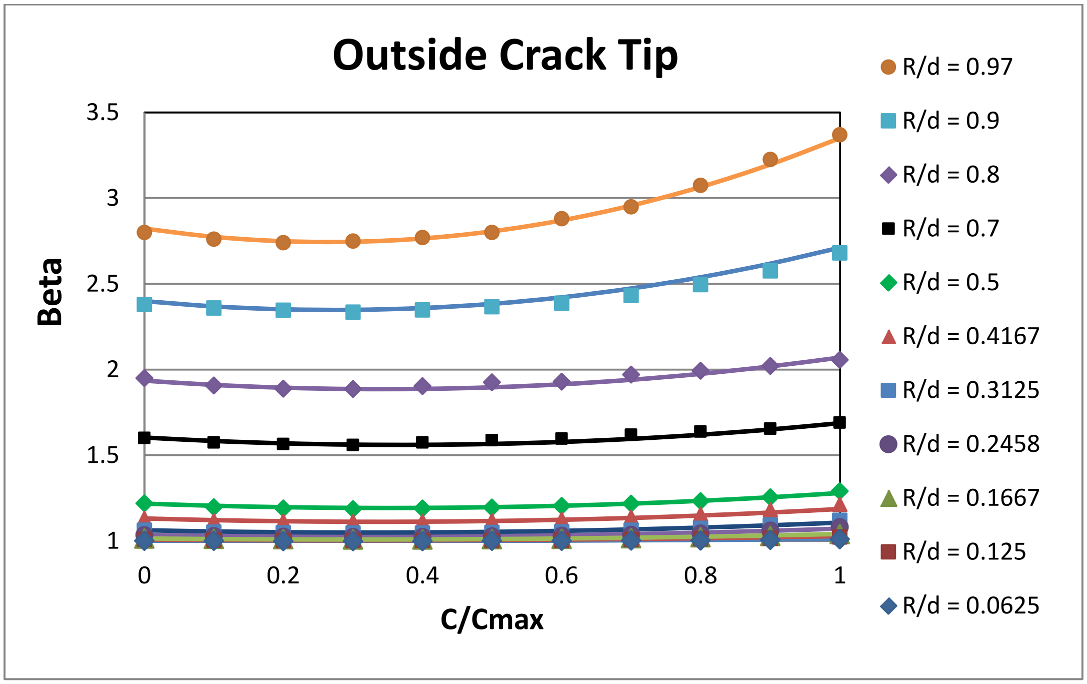

Figure 13: Beta Correction for the Outside Crack Tip

Additional StressCheck results for several crack lengths and hole spacing's were also compared to the new closed-form solution as indicated in Figures 14 and 15.

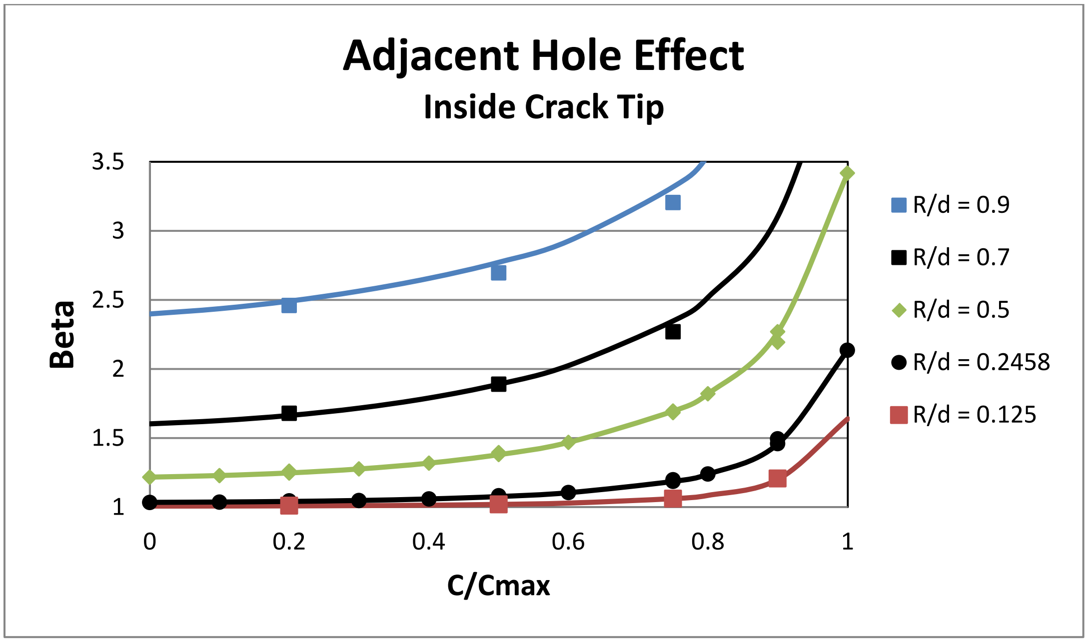

Figure 14: StressCheck Results for the Inside Crack Tip

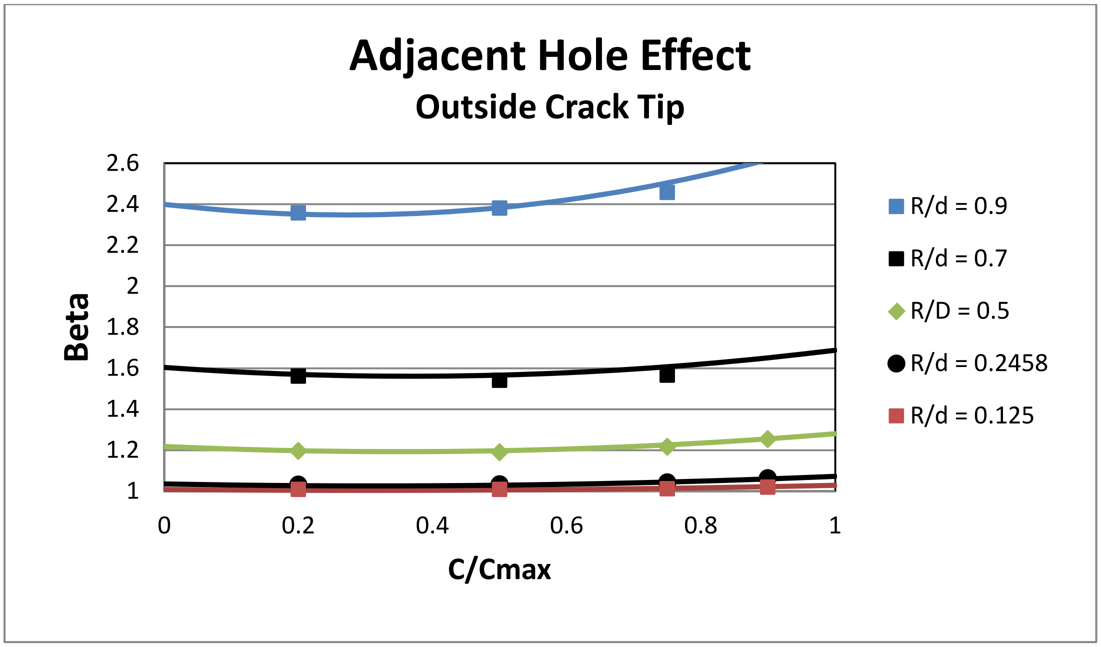

Figure 15**: StressCheck Results for the Outside Crack Tip**

There is very good agreement between the new FE results and the closed-form solutions. At the higher values of R/d and longer crack lengths, the solutions appears to be somewhat conservative. The largest difference seen was just over 3 percent.

2 Summary

The closed-form solutions documented here may be useful in the development of a new, general closed-form solution for the MSD problem. The accuracy of the solutions are within approximately 3% of all of the detailed FE analysis results, and within 1% of the majority of the results for all of the cases presented in this report. The finite element results used to develop these new solutions are provided in Appendix D.

Care was taken to use well behaved functions as far as possible to fit the data. The bounds of the parameters were selected to cover the most practical cases, but it seems reasonable to expect that the solutions could be used beyond the limits shown in this report (except where clearly indicated). However, the accuracy of any extrapolated solution would, of course, be more uncertain as the parameters increasingly exceed the limits.

Finally, while the method of compounding can be a very powerful tool, it should not be used to combine cases with different boundary conditions. For example, the holes modeled in this report were all open and un-filled. The solutions provided in this report should not be expected to apply to the case of a filled and/or a pin loaded hole. Additional work is required to characterize the behavior of filled and pin loaded holes as part of a complete MSD closed-form K-solution.

3 References

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Appendix A: Closed-Form Beta Correction for Two, Un-Equal Through Cracks on Either Side of an Open Hole

Beta Correction for C_p

$$F_{Cp} = \lambda_0 + \lambda_1\left(\frac{C_s}{R}\right) + \lambda_2\left(\frac{C_s}{R}\right)^2 + \lambda_3\left(2^{-20\left(\frac{C_s}{R}\right)}\right)$$

$$\lambda_0 = 0.97 + 0.03\left(2^{-\left(\frac{C_p}{R}\right)}\right)$$

$$\lambda_1 = 0.006 + 0.0325\left(1.6^{-0.95\left(\frac{C_p}{R}\right)}\right)$$

$$\lambda_2 = -0.0005 + 0.000475\left(1.65^{-1.125\left(\frac{C_p}{R}\right)}\right)$$

$$\lambda_3 = 0.0089 + 0.012\left(1.45^{-2\left(\frac{C_p}{R}\right)}\right)$$

Where: $C_s \le C_p$

Beta Correction for C_s

$$F_{Cs} = \lambda_0 + \lambda_1\left(\frac{C_s}{R}\right) + \lambda_2\left(\frac{C_s}{R}\right)^2 + \lambda_3\left(\frac{C_s}{R}\right)^3 + \lambda_4\left(\frac{C_s}{R}\right)^4 + \lambda_5\left(\frac{C_s}{R}\right)^5 + \lambda_6\left(\frac{C_s}{R}\right)^6$$

$$\lambda_0 = 0.9733 + 0.0292\left(\frac{C_P}{R}\right) - 0.0062\left(\frac{C_P}{R}\right)^2 + 0.00044\left(\frac{C_P}{R}\right)^3$$

$$\lambda_1 = 0.071 + 0.0219\left(\frac{C_P}{R}\right) + 0.00385\left(\frac{C_P}{R}\right)^2 - 0.000634\left(\frac{C_P}{R}\right)^3$$

$$\lambda_2 = -0.068 - 0.015\left(\frac{C_P}{R}\right) - 0.0086\left(\frac{C_P}{R}\right)^2 + 0.0012205\left(\frac{C_P}{R}\right)^3$$

$$\lambda_3 = 0.0003 - 0.009 \left(\frac{C_P}{R}\right)^2 + 0.016 \left(\frac{C_P}{R}\right)^3 - 0.00468 \left(\frac{C_P}{R}\right)^4$$

$$-0.00038825 \left(\frac{C_P}{R}\right)^5$$

$$\lambda_4 = 0.001143 + 0.034 \left(\frac{C_P}{R}\right) - 0.02655 \left(\frac{C_P}{R}\right)^2 + 0.004915 \left(\frac{C_P}{R}\right)^3$$

$$-0.0001357 \left(\frac{C_P}{R}\right)^4 - 0.00000075 \left(\frac{C_P}{R}\right)^5 - 0.0000005293 \left(\frac{C_P}{R}\right)^7$$

$$\lambda_5 = -0.001126 - 0.00254 \left(\frac{C_P}{R}\right) + 0.00254 \left(\frac{C_P}{R}\right)^2 - 0.0005321 \left(\frac{C_P}{R}\right)^3$$

$$+0.0000202 \left(\frac{C_P}{R}\right)^4 + 0.00000033836 \left(\frac{C_P}{R}\right)^6$$

$$\lambda_6 = 0.000065 + 0.000094959 \left(\frac{C_P}{R}\right) - 0.000054823 \left(\frac{C_P}{R}\right)^2$$

$$+0.000005383 \left(\frac{C_P}{R}\right)^3 + 0.00000001055761 \left(\frac{C_P}{R}\right)^5$$

Where: $C_s \leq C_p$

Appendix B: Closed-Form Beta Solution for Two Through Cracks Along a Plane in an Infinite Plate

Inside Crack Tip

$$\boldsymbol{\beta_{11}} = \frac{1 + \lambda_1\left(\frac{C_1 + C_2}{d}\right) + \lambda_2\left(\frac{C_1 + C_2}{d}\right)^3 + \lambda_2\left(\frac{C_1 + C_2}{d}\right)^6 + \lambda_3\left(\frac{C_1 + C_2}{d}\right)^{12}}{\left(1 - \left(\frac{C_1 + C_2}{d}\right)^{40}\right)^{0.03}}$$

$$\lambda_1 = 0.1\left(1.095 + 1.4\left(\frac{C_1}{C_2}\right)^{-0.575}\right)^{-1.2\left(\frac{C_1}{C_2}\right)}$$

$$\lambda_2 = 0.39\left(3^{-2.25\left(\frac{C_1}{C_2}\right)}\right) + 0.3\left(1.15^{-2.55\left(\frac{C_1}{C_2}\right)}\right)$$

$$\lambda_3 = 0.01 + 0.25\left(3^{-1.75\left(\frac{C_1}{C_2}\right)}\right) + 0.35\left(1.4^{-0.25\left(\frac{C_1}{C_2}\right)}\right)$$

Outside Crack Tip

$$\boldsymbol{\beta_{12}} = \frac{1 + \lambda_1\left(\frac{C_1 + C_2}{d}\right) + \lambda_2\left(\frac{C_1 + C_2}{d}\right)^2 + \lambda_3\left(\frac{C_1 + C_2}{d}\right)^7}{\left(1 - \left(\frac{C_1 + C_2}{d}\right)^{40}\right)^{0.01}}$$

$$\lambda_1 = 0.007\left(2.5^{-2\left(\frac{C_1}{C_2}\right)}\right) + 0.006\left(2.5^{-0.225\left(\frac{C_1}{C_2}\right)}\right)$$

$$\boldsymbol{\lambda_2 = 0.44\left(2.75^{-3.2\left(\frac{C_1}{C_2}\right)}\right) + 0.1\left(3.5^{-0.25\left(\frac{C_1}{C_2}\right)}\right)}$$

$$\boldsymbol{\lambda_3 = 1.055\left(5^{-3.45\left(\frac{C_1}{C_2}\right)}\right) + 0.1\left(1.14^{-4\left(\frac{C_1}{C_2}\right)}\right)}$$

Appendix C: Closed-Form Beta Solution for a Through Crack Approaching an Open Hole in an Infinite Plate

Inside Crack Tip

$$\boldsymbol{\beta_{inside} = \lambda_0 + \lambda_1\left(\frac{C}{C_{max}}\right) + \lambda_2\left(\frac{C}{C_{max}}\right)^2 + \lambda_3\left(\frac{C}{C_{max}}\right)^4 + \lambda_4\left(\frac{C}{C_{max}}\right)^8 + \lambda_5\left(\frac{C}{C_{max}}\right)^{14} + \lambda_6\left(\frac{C}{C_{max}}\right)^{24}}$$

$$\boldsymbol{\lambda_0 = 1 + 0.007\left(\frac{R}{d}\right) + 0.39\left(\frac{R}{d}\right)^2 + 0.46\left(\frac{R}{d}\right)^3 + 0.725\left(\frac{R}{d}\right)^4 + 0.45\left(\frac{R}{d}\right)^5}$$

$$\boldsymbol{\lambda_1 = 0.007\left(\frac{R}{d}\right) + 0.243\left(\frac{R}{d}\right)^2 + 0.13\left(\frac{R}{d}\right)^3}$$

$$\boldsymbol{\lambda_2 = 1.625\left(\frac{R}{d}\right)^{2.25} - 0.95\left(\frac{R}{d}\right)^7}$$

$$\boldsymbol{\lambda_3 = 0.5\left(\frac{R}{d}\right) + 2\left(\frac{R}{d}\right)^2 + 1.475\left(\frac{R}{d}\right)^3 - 5.3\left(\frac{R}{d}\right)^4 + 0.4\left(\frac{R}{d}\right)^7 + 1.2\left(\frac{R}{d}\right)^{12}}$$

$$\boldsymbol{\lambda_4 = 0.4\left(\frac{R}{d}\right) + 2.875\left(\frac{R}{d}\right)^9}$$

$$\boldsymbol{\lambda_5 = \left(0.2\ \ln\left[\left(\frac{R}{d}\right)^{0.2}\right] + 1.275\right)\left(1 - e^{-3\left(\frac{R}{d}\right)}\right)}$$

$$\boldsymbol{\lambda_6 = 0.275\left[3.75^{\left(-6.5\left(\frac{R}{d}\right)\right)}\right]}$$

Outside Crack Tip

$$\boldsymbol{\beta_{outside} = \lambda_0 + \lambda_1 \left(\frac{C}{C_{max}}\right) + \lambda_2 \left(\frac{C}{C_{max}}\right)^2}$$

$$\boldsymbol{\lambda_0 = 1 + 0.007 \left(\frac{R}{d}\right) + 0.39 \left(\frac{R}{d}\right)^2 + 0.46 \left(\frac{R}{d}\right)^3 + 0.725 \left(\frac{R}{d}\right)^4 + 0.45 \left(\frac{R}{d}\right)^5}$$

$$\boldsymbol{\lambda_1 = -0.225 \left(\frac{R}{d}\right) - 0.1 \left(\frac{R}{d}\right)^2 - 0.11 \left(\frac{R}{d}\right)^4 - 0.45 \left(\frac{R}{d}\right)^{30}}$$

$$\boldsymbol{\lambda_2 = 0.4 \left(\frac{R}{d}\right) + 0.1 \left(\frac{R}{d}\right)^4 + 0.925 \left(\frac{R}{d}\right)^{12}}$$

Appendix D: Finite Element Data

Two, Through Cracks at an Open Hole in a Wide Plate

W/D	D				
100	0.25				
C1	**C2**	**K1**	**K2**	**Beta1**	**Beta 2**
0.0000	0.0000			3.3650	3.3650
0.0050	0.0050	0.3885	0.3885	3.0998	3.0998
0.0100	0.0050	0.5096	0.3895	2.8751	3.1078
0.0200	0.0050	0.6315	0.3925	2.5193	3.1317
0.0300	0.0050	0.6917	0.3964	2.2531	3.1628
0.0400	0.0050	0.7261	0.4009	2.0483	3.1987
0.0500	0.0050	0.7482	0.4056	1.8878	3.2362
0.0500	0.0075	0.7490	0.4781	1.8898	3.1147
0.0500	0.0100	0.7500	0.5321	1.8923	3.0021
0.0500	0.0200	0.7559	0.6596	1.9072	2.6314
0.0500	0.0300	0.7636	0.7228	1.9267	2.3544
0.0500	0.0400	0.7722	0.7586	1.9484	2.1400
0.0500	0.0500	0.7815	0.7815	1.9718	1.9718
0.1000	0.0050	0.8006	0.4304	1.4284	3.4341
0.1000	0.0100	0.8025	0.5648	1.4318	3.1865
0.1000	0.0200	0.8083	0.7004	1.4421	2.7942
0.1000	0.0300	0.8160	0.7674	1.4558	2.4997
0.1000	0.0500	0.8341	0.8294	1.4881	2.0927
0.1000	0.0750	0.8582	0.8625	1.5311	1.7769
0.1000	0.1000	0.8824	0.8824	1.5743	1.5743
0.2500	0.0050	0.9142	0.4979	1.0316	3.9727
0.2500	0.0100	0.9158	0.6537	1.0334	3.6881
0.2500	0.0200	0.9208	0.8112	1.0390	3.2362
0.2500	0.0300	0.9274	0.8890	1.0465	2.8958
0.2500	0.0500	0.9428	0.9600	1.0638	2.4222
0.2500	0.0750	0.9634	0.9958	1.0871	2.0515
0.2500	0.1000	0.9848	1.0150	1.1112	1.8109
0.2500	0.1500	1.0260	1.0440	1.1577	1.5208
0.2500	0.2000	1.0660	1.0730	1.2029	1.3537
0.2500	0.2500	1.1040	1.1040	1.2457	1.2457
0.3750	0.0050	1.0100	0.5463	0.9305	4.3588
0.3750	0.0100	1.0110	0.7174	0.9315	4.0475
0.3750	0.0200	1.0160	0.8905	0.9361	3.5526
0.3750	0.0300	1.0220	0.9762	0.9416	3.1798

0.3750	0.0500	1.0350	1.0540	0.9536	2.6594
0.3750	0.0750	1.0540	1.0920	0.9711	2.2497
0.3750	0.1000	1.0730	1.1120	0.9886	1.9839
0.3750	0.1500	1.1110	1.1390	1.0236	1.6592
0.3750	0.2000	1.1470	1.1650	1.0568	1.4697
0.3750	0.2500	1.1820	1.1920	1.0890	1.3450
0.3750	0.3000	1.2150	1.2200	1.1194	1.2567
0.3750	0.3750	1.2630	1.2630	1.1636	1.1636
0.5000	0.0050	1.1010	0.5897	0.8785	4.7051
0.5000	0.0100	1.1020	0.7744	0.8793	4.3691
0.5000	0.0200	1.1060	0.9610	0.8825	3.8338
0.5000	0.0300	1.1110	1.0540	0.8864	3.4332
0.5000	0.0500	1.1230	1.1390	0.8960	2.8738
0.5000	0.0750	1.1400	1.1790	0.9096	2.4289
0.5000	0.1000	1.1570	1.1990	0.9232	2.1392
0.5000	0.1500	1.1930	1.2250	0.9519	1.7845
0.5000	0.2000	1.2250	1.2480	0.9774	1.5744
0.5000	0.3000	1.2890	1.3000	1.0285	1.3391
0.5000	0.4000	1.3500	1.3540	1.0771	1.2079
0.5000	0.5000	1.4080	1.4080	1.1234	1.1234
0.7500	0.0050	1.2650	0.6656	0.8241	5.3107
0.7500	0.0100	1.2660	0.8748	0.8248	4.9355
0.7500	0.0200	1.2690	1.0880	0.8267	4.3405
0.7500	0.0300	1.2740	1.1930	0.8300	3.8860
0.7500	0.0500	1.2850	1.2890	0.8371	3.2523
0.7500	0.0750	1.2990	1.3350	0.8463	2.7503
0.7500	0.1000	1.3160	1.3550	0.8573	2.4175
0.7500	0.2000	1.3740	1.4000	0.8951	1.7662
0.7500	0.3000	1.4310	1.4450	0.9323	1.4884
0.7500	0.4000	1.4850	1.4930	0.9674	1.3318
0.7500	0.5000	1.5380	1.5420	1.0020	1.2303
0.7500	0.6000	1.5880	1.5900	1.0345	1.1581
0.7500	0.7000	1.6380	1.6380	1.0671	1.1046
0.7500	0.7500	1.6610	1.6610	1.0821	1.0821

W/D	D				
100	0.5				
C1	**C2**	**K1**	**K2**	**Beta1**	**Beta 2**
0.0100	0.0100	0.5495	0.5495	3.1002	3.1002
0.0500	0.0100	0.9427	0.5579	2.3786	3.1476
0.0500	0.0200	0.9445	0.7318	2.3831	2.9195
0.0500	0.0300	0.9484	0.8365	2.3929	2.7248
0.0500	0.0400	0.9525	0.9068	2.4033	2.5580
0.0500	0.0500	0.9567	0.9567	2.4139	2.4139
0.5000	0.1000	1.3330	1.3580	1.0636	2.4228
0.5000	0.2000	1.3920	1.4350	1.1107	1.8103
0.5000	0.3000	1.4500	1.4750	1.1569	1.5193
0.5000	0.5000	1.5590	1.5590	1.2439	1.2439
1.0000	0.5000	1.7790	1.8000	1.0037	1.4362
1.0000	1.0000	1.9900	1.9900	1.1227	1.1227

Two, Through Cracks in a Wide Plate

				Internal Tips				Internal Tips		
C1	**C2**	**d**	**K1**	**K2**	**K3**	**K4**	**Beta 1**	**Beta 2**	**Beta 3**	**Beta 4**
0.05	0.05	2	0.3964	0.3965	0.3965	0.3964	1.0002	1.0004	1.0004	1.0002
0.1	0.05	2	0.5608	0.5609	0.3968	0.3968	1.0005	1.0007	1.0012	1.0012
0.2	0.05	2	0.7937	0.7946	0.3984	0.3983	1.0013	1.0024	1.0052	1.0050
0.2	0.1	2	0.7947	0.7955	0.5638	0.5634	1.0026	1.0036	1.0059	1.0052
0.2	0.2	2	0.7975	0.7990	0.7990	0.7975	1.0061	1.0080	1.0080	1.0061
0.3	0.3	2	0.9810	0.9843	0.9843	0.9810	1.0105	1.0139	1.0139	1.0105
0.4	0.3	2	1.1320	1.1380	0.9952	0.9889	1.0098	1.0152	1.0251	1.0186
0.4	0.4	2	1.1410	1.1510	1.1510	1.1410	1.0178	1.0268	1.0268	1.0178
0.5	0.3	2	1.2650	1.2740	1.0100	0.9990	1.0093	1.0165	1.0404	1.0290
0.5	0.4	2	1.2750	1.2910	1.1700	1.1530	1.0173	1.0301	1.0437	1.0285
0.5	0.5	2	1.2880	1.3140	1.3140	1.2880	1.0277	1.0484	1.0484	1.0277
0.6	0.6	2	1.4290	1.4860	1.4860	1.4290	1.0408	1.0824	1.0824	1.0408
0.75	0.25	2	1.5460	1.5680	0.9705	0.9495	1.0072	1.0215	1.0951	1.0714
0.75	0.5	2	1.5780	1.6390	1.4040	1.3370	1.0280	1.0678	1.1202	1.0668
0.75	0.75	2	1.6390	1.8120	1.8120	1.6390	1.0678	1.1805	1.1805	1.0678
0.9	0.3	2	1.7010	1.7430	1.1180	1.0740	1.0116	1.0366	1.1516	1.1063
0.9	0.45	2	1.7230	1.8000	1.4020	1.3110	1.0247	1.0705	1.1791	1.1026
0.9	0.9	2	1.8790	2.4450	2.4450	1.8790	1.1175	1.4541	1.4541	1.1175
1	0.5	2	1.8310	1.9720	1.5760	1.4250	1.0330	1.1126	1.2575	1.1370
1.2	0.4	2	2.0060	2.1620	1.5630	1.3650	1.0332	1.1135	1.3943	1.2177

1.2	0.3	2	1.9700	2.0400	1.2970	1.1820	1.0146	1.0507	1.3360	1.2175
1.4	0.2	2	2.1090	2.1720	1.1900	1.0740	1.0056	1.0357	1.5013	1.3549
1.5	0.1	2	2.1730	2.1960	0.8821	0.8250	1.0010	1.0116	1.5738	1.4719
0.05	0.05	1.5	0.3965	0.3966	0.3965	0.3966	1.0004	1.0007	1.0004	1.0007
0.1	0.05	1.5	0.5610	0.5614	0.3973	0.3972	1.0009	1.0016	1.0024	1.0022
0.2	0.05	1.5	0.7937	0.7945	0.4000	0.3998	1.0013	1.0023	1.0093	1.0087
0.2	0.1	1.5	0.7947	0.7949	0.5659	0.5653	1.0026	1.0028	1.0096	1.0086
0.2	0.2	1.5	0.7994	0.8012	0.8012	0.7994	1.0085	1.0108	1.0108	1.0085
0.3	0.3	1.5	0.9879	0.9971	0.9971	0.9879	1.0176	1.0271	1.0271	1.0176
0.4	0.2	1.5	1.1320	1.1370	0.8282	0.8197	1.0098	1.0143	1.0448	1.0341
0.4	0.4	1.5	1.1590	1.1860	1.1860	1.1590	1.0339	1.0580	1.0580	1.0339
0.5	0.5	1.5	1.3150	1.3980	1.3980	1.3150	1.0492	1.1154	1.1154	1.0492
0.6	0.2	1.5	1.3820	1.4050	0.8806	0.8577	1.0066	1.0234	1.1109	1.0820
0.6	0.3	1.5	1.3950	1.4390	1.0930	1.0470	1.0161	1.0481	1.1259	1.0785
0.6	0.4	1.5	1.4150	1.4910	1.2870	1.2110	1.0306	1.0860	1.1481	1.0803
0.6	0.6	1.5	1.4820	1.6940	1.6940	1.4820	1.0794	1.2339	1.2339	1.0794
0.05	0.05	1	0.3968	0.3969	0.3969	0.3968	1.0012	1.0014	1.0014	1.0012
0.1	0.05	1	0.5616	0.5617	0.3984	0.3982	1.0020	1.0021	1.0052	1.0047
0.1	0.1	1	0.5635	0.5638	0.5638	0.5635	1.0054	1.0059	1.0059	1.0054
0.181	0.0181	1	0.7538	0.7546	0.2425	0.2424	0.9996	1.0007	1.0169	1.0165
0.2	0.05	1	0.7941	0.7953	0.4050	0.4041	1.0018	1.0033	1.0219	1.0196
0.2	0.1	1	0.7968	0.7994	0.5737	0.5713	1.0052	1.0085	1.0236	1.0193
0.2	0.2	1	0.8071	0.8143	0.8143	0.8071	1.0182	1.0273	1.0273	1.0182
0.3	0.1	1	0.9747	0.9785	0.5912	0.5853	1.0040	1.0079	1.0548	1.0442
0.3	0.2	1	0.9875	1.0020	0.8441	0.8260	1.0172	1.0321	1.0649	1.0421
0.3	0.3	1	1.0100	1.0480	1.0480	1.0100	1.0404	1.0795	1.0795	1.0404
0.364	0.0364	1	1.0700	1.0710	0.3642	0.3620	1.0006	1.0015	1.0770	1.0705
0.38	0.019	1	1.0930	1.0930	0.2645	0.2637	1.0004	1.0004	1.0826	1.0793
0.4	0.2	1	1.1370	1.1720	0.8920	0.8546	1.0143	1.0455	1.1253	1.0781
0.4	0.4	1	1.2070	1.3800	1.3800	1.2070	1.0767	1.2310	1.2310	1.0767
0.45	0.45	1	1.3290	1.7320	1.7320	1.3290	1.1177	1.4567	1.4567	1.1177
0.5	0.25	1	1.2910	1.3790	1.1130	1.0030	1.0301	1.1003	1.2559	1.1318
0.6	0.3	1	1.4500	1.7800	1.5780	1.1900	1.0561	1.2965	1.6254	1.2258
0.545	0.0545	1	1.3100	1.3170	0.4999	0.4880	1.0011	1.0065	1.2081	1.1794
0.574	0.0287	1	1.3440	1.3500	0.3692	0.3640	1.0008	1.0053	1.2295	1.2122
0.72	0.18	1	1.5450	1.7600	1.3490	1.0420	1.0273	1.1702	1.7939	1.3857
0.73	0.073	1	1.5260	1.5480	0.7412	0.6798	1.0077	1.0222	1.5477	1.4195
0.76	0.038	1	1.5510	1.5650	0.5500	0.5200	1.0038	1.0128	1.5918	1.5050
0.82	0.082	1	1.6200	1.7120	1.0180	0.8432	1.0093	1.0666	2.0057	1.6613
0.05	0.05	0.75	0.3972	0.3973	0.3973	0.3972	1.0022	1.0024	1.0024	1.0022
0.1	0.05	0.75	0.5620	0.5624	0.4002	0.3997	1.0027	1.0034	1.0098	1.0085
0.1	0.1	0.75	0.5654	0.5667	0.5667	0.5654	1.0087	1.0111	1.0111	1.0087
0.2	0.05	0.75	0.7943	0.7965	0.4124	0.4103	1.0021	1.0048	1.0405	1.0352

0.2	0.1	0.75	0.7989	0.8040	0.5855	0.5794	1.0079	1.0143	1.0446	1.0337
0.2	0.2	0.75	0.8184	0.8388	0.8388	0.8184	1.0325	1.0582	1.0582	1.0325
0.3	0.2	0.75	1.0020	1.0490	0.9093	0.8543	1.0321	1.0805	1.1471	1.0778
0.3	0.3	0.75	1.0490	1.1930	1.1930	1.0490	1.0805	1.2289	1.2289	1.0805
0.4	0.2	0.75	1.1630	1.2750	1.0570	0.9172	1.0375	1.1374	1.3335	1.1571
0.5	0.1	0.75	1.2670	1.3270	0.8205	0.7286	1.0109	1.0588	1.4639	1.2999
0.55	0.05	0.75	1.3210	1.3470	0.6141	0.5669	1.0050	1.0247	1.5495	1.4304
0.05	0.05	0.5	0.3982	0.3986	0.3986	0.3982	1.0047	1.0057	1.0057	1.0047
0.1	0.05	0.5	0.5630	0.5641	0.4056	0.4039	1.0045	1.0064	1.0234	1.0191
0.2	0.05	0.5	0.7965	0.8008	0.4377	0.4293	1.0048	1.0103	1.1044	1.0832
0.2	0.1	0.5	0.8068	0.8266	0.6309	0.6046	1.0178	1.0428	1.1256	1.0787
0.2	0.2	0.5	0.8570	0.9742	0.9742	0.8570	1.0812	1.2290	1.2290	1.0812
0.05	0.05	0.375	0.3995	0.4005	0.4005	0.3995	1.0080	1.0105	1.0105	1.0080
0.1	0.05	0.375	0.5648	0.5679	0.4140	0.4097	1.0077	1.0132	1.0446	1.0337
0.15	0.15	0.375	0.7422	0.8437	0.8437	0.7422	1.0812	1.2290	1.2290	1.0812
0.2	0.05	0.375	0.8015	0.8169	0.4867	0.4601	1.0111	1.0306	1.2280	1.1609
0.2	0.1	0.375	0.8216	0.9013	0.7471	0.6485	1.0365	1.1370	1.3329	1.1570

Three, Through Cracks in a Wide Plate (results reported for the center crack, C1**)**

C1	C2	d1	d2	K11	K12	Beta 11	Beta 12
0.2	0.2	1	1	0.8282	0.8282	1.0448	1.0448
0.25	0.25	1	1	0.9542	0.9542	1.0767	1.0767
0.35	0.35	1	1	1.2570	1.2570	1.1987	1.1987
0.2	0.2	1	1.5	0.8212	0.8130	1.0360	1.0257
0.25	0.25	1	1.5	0.9399	0.9250	1.0606	1.0438
0.35	0.35	1	1.5	1.2190	1.1540	1.1625	1.1005
0.2	0.1	1	1	0.8016	0.8016	1.0113	1.0113
0.25	0.125	1	1	0.9019	0.9019	1.0177	1.0177
0.35	0.175	1	1	1.0950	1.0950	1.0443	1.0443
0.5	0.25	1	1	1.4220	1.4220	1.1346	1.1346
0.2	0.1	1	1.5	0.7999	0.7991	1.0091	1.0081
0.25	0.125	1	1.5	0.8989	0.8964	1.0143	1.0115
0.35	0.175	1	1.5	1.0870	1.0720	1.0366	1.0223
0.5	0.25	1	1.5	1.3970	1.3280	1.1146	1.0596
0.1	0.2	1	1	0.5838	0.5838	1.0416	1.0416
0.125	0.25	1	1	0.6693	0.6693	1.0680	1.0680
0.175	0.35	1	1	0.8517	0.8517	1.1487	1.1487
0.25	0.5	1	1	1.2480	1.2480	1.4082	1.4082
0.1	0.2	1	1.5	0.5783	0.5752	1.0318	1.0262
0.125	0.25	1	1.5	0.6592	0.6536	1.0519	1.0430
0.175	0.35	1	1.5	0.8260	0.8087	1.1140	1.0907
0.25	0.5	1	1.5	1.1680	1.0760	1.3179	1.2141

Through Crack Approaching an Open Hole in a Wide Plate

	Inside Crack Tip Beta-Values				
	R/d				
C/Cmax	**0.125**	**0.2458**	**0.5**	**0.7**	**0.9**
0.2	1.0107	1.0450	1.2614	1.6810	2.4600
0.5	1.0210	1.0821	1.3947	1.8901	2.6966
0.75	1.0614	1.1981	1.6851	2.2706	3.2062
0.9	1.2073	1.4942	2.1934		

	Outside Crack Tip Beta-Values				
	R/d				
C/Cmax	**0.125**	**0.2458**	**0.5**	**0.7**	**0.9**
0.2	1.0077	1.0318	1.1961	1.5616	2.3572
0.5	1.0082	1.0325	1.1886	1.5399	2.3811
0.75	1.0112	1.0425	1.2152	1.5650	2.4572
0.9	1.0198	1.0621	1.2532		